BCA–105(N)

B. C. A. (First Semester)
EXAMINATION, Dec., 2019

(New Course)

Paper Fifth

MATHEMATICS—I

Time : Three Hours] *[Maximum Marks : 7*

Note : Attempt questions from all Sections as directed.

Inst. : The candidates are required to answer only in serial order. If there are many parts of a question, answer them in continuation.

Section—A

(Short Answer Type Questions)

Note : Attempt all questions. Each question carries 3 marks.

1. (A) Using Cramer's rule, solve the following system of equation :

$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$

(B) Find the value of :

$$\int \frac{\sin^4 x}{\cos^2 x} dx$$

(C) Evaluate the following :

(i) $\lim_{x \to 0} \frac{(1+x)^6 - 1}{(1+x)^2 - 1}$

(ii) $\lim_{x \to 0} \frac{\log x}{x}$

(D) Find C of the mean value theorem, when :

$$f(x) = x^3 - 3x - 2$$

in [–2, 3].

(E) Find the rank of the matrix :

$$A = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix}$$

(F) Find the maximum and minimum value of the function :

$$f(x) = x^3 - 3x^2 + 6$$

(G) Evaluate :

$$\int_0^1 x^4 (1 - \sqrt{x})^5 dx$$

(H) If :

$$\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$$

and $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$

find the value of :

$$(\vec{a} + 4\vec{b}).(2\vec{a} - \vec{b})$$

(I) Differentiate the following function w.r.t. x :

(i) $\dfrac{\sin x - x\cos x}{x\sin x + \cos x}$

(ii) $(\log x)^{\sin x}$

Section—B

(Long Answer Type Questions)

Note : Attempt any *two* questions. Each question carries 12 marks.

2. Show that the matrix :

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

satisfies its own characteristic equation, find A^{-1}.

3. Prove that the area of the triangle whose vertices are $\vec{A}, \vec{B}, \vec{C}$ is :

$$\frac{1}{2}\left[\vec{B}\times\vec{C} + \vec{C}\times\vec{A} + \vec{A}\times\vec{B}\right]$$

4. If : $y = \left[x + \sqrt{x^2+1}\right]^m$

prove that :

(i) $(x^2+1)y_2 + xy_1 - m^2 y = 0$

(ii) $y_{n+2} + (n^2 - m^2)y_n = 0$ at $x = 0$

5. By using Maclaurin's theorem, find the first four terms in the expansion of :

$$\log(1 + \tan x)$$

in the power of x.

Section—C

(Long Answer Type Questions)

Note : Attempt any *two* questions. Each question carries 12 marks.

6. Determine the eigen value and the eigen vector of the matrix :

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

7. Differentiate the following function w.r.t. x and find nth derivative :

(i) $\cos x \cos 2x \cos 3x$

(ii) $e^{2x}\cos^2 x \sin x$

(iii) $\dfrac{1}{1 + x + x^2 + x^3}$

8. (a) Show that the function defined by :

$$f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$$

is continuous at $x = 3$.

(b) Find the values of a and b such that :

$$\lim_{x \to 0} \frac{x(a + b\cos x) - c\sin x}{x^5} = 1$$

9. Express :

$$\begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ca - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix}$$

as the square of a determinant and hence find its value. 4,000